



# Review of Ecrits Logiques by Jacques Herbrand

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## BOOK REVIEWS

*ECRITS LOGIQUES*. By JACQUES HERBRAND, edited by JEAN VAN HEIJENOORT. Paris, Presses Universitaires de France, 1968. Pp. 244. 24f.

In the years 1928-1931, Jacques Herbrand developed an approach to certain questions of mathematical logic that has greatly influenced many subsequent developments in this field. Although he was primarily a mathematician (indeed, he is also known for his very important work in algebra), his interest and goals in studying logic were based on a sensitivity to problems in the philosophy of mathematics. The present volume comprises all of his writings on logic, including one previously unpublished summary of his work, plus several brief explanatory and biographical notes by the editor, Jean van Heijenoort, and by Claude Chevalley and Albert Lautman.

Herbrand's interest in logic developed from reading the *Principia Mathematica*. For Herbrand, this work furnished empirical evidence that mathematics could be formalized using relatively few primitive symbols, axioms, and rules, and hence showed that the formalism of Russell and Whitehead did in fact capture the intuitive notions of mathematics. However, he rejected both the philosophical view that this furnished a "reduction" of mathematics to logic and the Platonist ontology of propositional functions or classes. Rather, Herbrand adopted the position of the Hilbert school: such formalization shows that mathematics can be considered as rules for the manipulation of combinations of signs, and that the logician's task is the metamathematical one of studying these rules. Metamathematics, as opposed to mathematics, has content; hence, the methods used in this study must allow no doubt as to their validity. His explanation of finitism, in "Les bases de la logique hilbertienne," relies on intuitive, common-sense notions of rigor and certainty, and ignores the more subtle issues raised by Hilbert and Brouwer. His formulation, however, makes clear the assumptions about these notions necessary to the finitist viewpoint, and avoids the philosophical preconceptions underlying the thought of the latter two men. In addition, the simplicity of his exposition occasionally enables Herbrand to give a most perspicuous statement to certain issues, such as the difference between mathematical and metamathematical induction that makes the latter proof-theoretically acceptable while the former is not.

In his technical work, Herbrand added to the finitist program in a most important way. Instead of considering separately the problems regarding decision procedures and consistency, he sought certain canonical syntactic properties of the formulas provable in a given

formal system. Results in this vein could then be applied to the two problems. His method here shows the other major influence on him, that of Löwenheim. Löwenheim, in the proof of the theorem bearing his name, used the idea of interpreting existential and universal quantifiers as infinite disjunctions and conjunctions. Thus, to every formula with quantifiers is associated an infinitely long quantifier-free expression. Herbrand adapted this notion to a finitist framework by considering provability, a proof-theoretic property, instead of validity, a model-theoretic one, and by dealing with finite initial segments of the infinitely long formulas. In his thesis, “Recherches sur la théorie de la démonstration,” these procedures are used to prove results of the following type: canonically generated sequences of quantifier-free expressions are associated with formulas of quantification theory such that some expression from every such sequence is truth-functionally valid if the formula is provable in quantification theory. Furthermore, from the proof of the original formula, the quantifier-free expression that is truth-functionally valid can be effectively obtained.

In this thesis and the subsequent papers. “Sur le problème fondamental de la logique mathématique” and “Sur la non-contradiction de l’arithmétique,” Herbrand applied these results to the decision and consistency problems of certain formalisms. As a consequence, he was able not only to obtain more powerful theorems than had previously been proven, but also to furnish a more unified and finitistic approach to these problems by considering the syntactic properties with which he had dealt in “Recherches.”

Unfortunately, these technical papers are densely written and contain numerous obscurities. Furthermore, there is a crucial error in “Recherches” which invalidates the proof that every provable formula has a truth-functionally valid Herbrand expansion. This error was not corrected until recently, by Burton Dreben, although weaker versions of the theorem were proved by other methods. The correction appears among the notes to the translation of Chapter 5 of “Recherches” in van Heijenoort’s *From Frege to Gödel: A Sourcebook in Mathematical Logic* (Cambridge, Mass., 1967). With regard to other difficulties in understanding Herbrand’s proof, van Heijenoort’s introduction to the *Ecrits* not only explains the error, but also gives a useful survey of Herbrand’s work and the subsequent history of studies of Herbrand’s method. The volume is not intended as a critical edition, however, and most of the specific obscurities are not elucidated. Hence, it is almost a necessity to read the above-mentioned notes and translation of Chapter 5 of “Recherches.”

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Herbrand's writings have influenced the subsequent development of logic in several ways. The strategy of finding cut-free proofs of the theorems of a formal system, which gained prominence through Gentzen's papers and lies at the heart of much work in proof theory, is in fact due to Herbrand; the Herbrand expansion method provides such proofs for theorems of quantification theory. Second, the idea of reducing quantified provable formulas in some manner to quantifier-free, finitistically provable ones formed the basis of the  $\epsilon$ -theorems of Hilbert and Bernays and, in turn inspired by these, the theory of interpretations as developed by Kreisel. More technically, Herbrand's specific ideas, in particular the use of Herbrand expansions, have been extended; a forthcoming monograph by Dreben, Denton, and Scanlon, *The Herbrand Theorem and the Consistency of Number Theory*, is a detailed study of this. Finally, Herbrand's method provides much material regarding proof procedures, elimination of quantifiers, and so on, and suggests questions which have not yet been explored.

WARREN D. GOLDFARB

*Harvard University*

**PHILOSOPHICAL PROBLEMS AND ARGUMENTS.** By JAMES W. CORNMAN and KEITH LEHRER. New York, The Macmillan Company, 1968. Pp. xii, 462. \$5.95.

Texts for introductory philosophy courses fall roughly into three classes: anthologies, introductory surveys, and classics. Each approach has its advantages and problems. In an anthology views are presented by the philosophers who really hold them. Students are introduced to thinkers they may study more deeply later. But the papers are often intended for a sophisticated audience; variations in style, terminology, and approach put irrelevant difficulties in the way of understanding. Classics have the advantage of being classics: they are worth reading for themselves, and not just as introductions to something else. But as introductions they are often one-sided. The philosophical level of introductory surveys is often low. Even the best are dull.

Cornman and Lehrer have written a text that has some of the advantages of anthologies and classics, and avoids some of the shortcomings of the usual introductory survey. The authors present the material dialectically, enabling them to state in a very thorough and generally fair way the main positions on each problem discussed, so the student